Teaching Plan / Lesson No.

Name of the Topic	Groups
Hours required	15
Learning Objectives	After completion of this topic, students will be able to define group and discuss some properties related to groups
Previous knowledge to be reminded	sets, functions
Examples / Illustrations	$(Z, +)$ is a group
Additional inputs	Applications of group theory
Teaching Aids used	Blackboard, Chalkpiece
References cited	Topics in Algebra By I.N. Herstein
Student Activity Planned after the teaching	Assignment, Seminar, slip test
Activity planned outside the class room, if any	
Any other activity	
Topic Synopsis	*(Continue on the reverse side if needed)* Def: Let G be a non-empty set and $*$ be a binary operation on G then $(G,*)$ is said to be a group if it satisfies the following 1. Associative property: $a*(b*c) = (a*b)*c \ \forall a,b,c \in G$ 2. Identity property: For each $a \in G$, there exists an element $e \in G$ such that $a*e = e*a = a$ 3. Inverse property: For each $a \in G$, there exists an element $b \in G$

such that $a*b = b*a = e$

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- In a group G, identity element is unique
- In a group G, inverse of any element is unique
- The set Q_+ of all +ve rational numbers forms an abelian group under the composition defined by $\circ$ such that $a \circ b = \frac{ab}{3}$ for $a, b \in Q_+$
- The set of matrices $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, $\alpha \in R$ forms a group w.r.t matrix multiplication if $\cos\theta = \cos\phi \Rightarrow \theta = \phi$
- The set Z of all integers form an abelian group w.r.t the operation defined by $a * b = a + b + 2$ for all $a, b \in Z$.
- Let $(G, \cdot)$ be an algebraic structure. Then $(G, \cdot)$ is a group iff (i) $a, b, c \in G \Rightarrow (ab)c = a(bc)$ (ii) $ax = b$, $ya = b$ have unique solutions in G for every $a, b \in G$.
- Show that the fourth roots of unity form an abelian w.r.t multiplication
- If every element of a group G is its own inverse, then $(G, \cdot)$ is an abelian group.
- Show that the set $G = \{ \dots -3m, -2m, -m, 0, m, 2m, 3m, \dots \}$ is an abelian group w.r.t usual addition, m being a fixed integer.

Teaching Plan / Lesson No.

Name of the Topic	Groups (Cont.)
Hours required	5
Learning Objectives	After completion of this topic, students will be able to use the addition and multiplication modulo m
Previous knowledge to be reminded	Groups
Examples / Illustrations	$Z_5 = \{0,1,2,3,4\}$ is an abelian group w.r.t addition modulo 5 $(+_5)$
Additional inputs	-
Teaching Aids used	Blackboard, Chalkpiece
References cited	Topics in Algebra by I.N. Herstein
Student Activity Planned after the teaching	Assignment, slip test
Activity planned outside the class room, if any	
Any other activity	
Topic Synopsis	*(Continue on the reverse side if needed)* Cancellation laws: Let G be a group. Then for $a,b,c \in G$, $ab = ac \Rightarrow b = c$ (left cancellation law) and $ba = ca \Rightarrow b = c$ (right cancellation law) * In a non-identity group G, for $a,b,x,y \in G$, the equations $ax = b$ and $ya = b$ have unique solutions

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* Let $(G, \cdot)$ be an algebraic structure. Then $(G, \cdot)$ is a group iff (i) $a, b, c \in G$ $\Rightarrow (ab)c = a(bc)$ (ii) $ax = b$, $ya = b$ have unique solutions in G for every $a, b \in G$.

* The set of n^{th} roots of unity under multiplication form a finite group

* The sets of all ordered pairs (a, b) of real numbers for which $a \neq 0$ w.r.t the operations $\times$ defined by $(a, b) \times (c, d) = (ac, bc + d)$ is a group.

* The fourth roots of unity forms an abelian group w.r.t multiplication.

* If every element of a group G is its own inverse, then $(G, \cdot)$ is an abelian group

Addition modulo m

Let $a, b \in Z$ and m be a fixed positive integer. If r is the remainder $(0 \leq r < m)$ when $a + b$ is divided by m, we define '$a +_m b$' $= r$ and we read '$a +_m b$' as 'a addition modulo m' b.

Ex: $20 +_6 5 = 1$, $-32 +_4 5 = 1$

Multiplication modulo m

If a and b are integers and p is a fixed positive integer and if ab is divided by p such that r is the remainder $(0 \leq r < p)$, we define $a \times_p b = r$. We read $a \times_p b$ as a 'multiplication modulo p' b.

Ex: $20 \times_6 5 = 4$, $2 \times_5 3 = 1$

* The set $G = \{0, 1, 2, \dots (m-1)\}$ of first m non-negative integers is an abelian group w.r.t the operation addition modulo m.

* The set of non-zero residue classes modulo a prime integer p forms an abelian group of order $p-1$ w.r.t multiplication of residue classes.

Teaching Plan / Lesson No.

Name of the Topic	order of an element of a group
Hours required	5
Learning Objectives	After completion of this topic, students will be able to find the order of any element of a finite group
Previous knowledge to be reminded	Groups
Examples / Illustrations	In Z_6, $o(2)=3$ $[3\cdot 2=0]$ In $G=\{1,-1\}$, $o(-1)=2$ $[(-1)^2=1]$
Additional inputs	-
Teaching Aids used	Blackboard, Chalkpiece
References cited	Topics in Algebra By I.N. Herstein
Student Activity Planned after the teaching	Assignment
Activity planned outside the class room, if any	-
Any other activity	-
Topic Synopsis	*(Continue on the reverse side if needed)* * Let G be a group and a be any element of G. Then the order of the element a is defined as the least positive integer n such that $a^n=e$ If there exists no positive integer n such that $a^n=e$, Then we say that a is of infinite order or zero order

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* The order of every element of a finite group is finite and less than or equal to the order of the group.

* In a group G, $o(a) = o(a^{-1})$ $\forall a \in G$

* If a is an element of a group G such that $o(a) = n$ Then $a^m = e$ iff n/m

* If a is an element of order n of a group G and $(p,n) = 1$ then $o(a^p) = n$

* Let G be an abelian group. If $a, b \in G$ such that $o(a) = m$, $o(b) = n$ and $(m,n) = 1$ Then $o(ab) = mn$.

* In a group G, if $a \in G$ and $o(a) = m$ then $o(a^k) = \frac{m}{(m,k)}$

* ~~In a~~ For any two elements $a, b \in G$, $o(a) = o(b^{-1}ab)$

* $Z_6 = \{0,1,2,3,4,5\}$

  $\cancel{o(1)} = 6 \cdot 1 = 0 \Rightarrow o(1) = 6$

  $o\cancel{(2)} = 3 \cdot 2 = 0 \Rightarrow o(2) = 3$

  $2 \cdot 3 = 0 \Rightarrow o(3) = 2$

  $3 \cdot 4 = 0 \Rightarrow o(4) = 3$

  $6 \cdot 5 = 0 \Rightarrow o(5) = 6$

  and $o(0) = 1$

* If every element of a group G except the identity element is of order two then the group is abelian.

Teaching Plan / Lesson No.

Name of the Topic	SUBGROUPS
Hours required	5
Learning Objectives	After completion of this topic, students will be able to find the subgroups ~~and~~ of a group and know some properties of subgroups
Previous knowledge to be reminded	Groups
Examples / Illustrations	$(\mathbb{Z}, +)$ is a subgroup of $(\mathbb{Q}, +)$ $(\mathbb{Z}, \cdot)$ is not a subgroup of $(\mathbb{Q}, \cdot)$ because $(\mathbb{Z}, \cdot)$ is not a group
Additional inputs	-
Teaching Aids used	Blackboard, Chalkpiece
References cited	Topics in Algebra By I.N. Herstein
Student Activity Planned after the teaching	Assignment, slip test
Activity planned outside the class room, if any	-
Any other activity	-
Topic Synopsis	*(Continue on the reverse side if needed)* Subgroup: Let G be a group and H be a non-empty subset of G. If H is also a group under the same operation as in G then H is called a subgroup of G * The identity of a subgroup H of a group is same as the identity of G.

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* If H is any subgroup of G then $H = H^{-1}$

* If H is any subgroup of a group G then $HH = H$

* A non-empty subset H of a group G is a subgroup of G iff

(i) $a, b \in H \Rightarrow ab \in H$ (ii) $a \in H \Rightarrow a^{-1} \in H$

* A non-empty subset H of a group G is a subgroup of G iff

$a, b \in H \Rightarrow ab^{-1} \in H$

* A necessary and sufficient condition for a non-empty subset of a group G to be a subgroup of G is that $HH^{-1} \subseteq H$

* A necessary and sufficient condition for a non-empty subset of a group G to be a subgroup of G is that $HH^{-1} = H$

* The necessary and sufficient condition for a finite subset H of a group G to be a subgroup of G is $a, b \in H \Rightarrow ab \in H$

Teaching Plan / Lesson No.

Name of the Topic	SUBGROUPS
Hours required	5
Learning Objectives	After completion of this topic, students will be able to discuss the algebra of subgroups.
Previous knowledge to be reminded	Subgroups
Examples / Illustrations	$H_1 = \{0,3\}$ and $H_2 = \{0,2,4\}$ are subgroups of $(\{0,1,2,3,4,5\}, +_6)$ $\Rightarrow H_1 \cap H_2 = \{0\}$ is a subgroup of Z_6
Additional inputs	
Teaching Aids used	Blackboard, Chalkpiece
References cited	Topics in Algebra By I.N. Herstein
Student Activity Planned after the teaching	Seminar
Activity planned outside the class room, if any	
Any other activity	
Topic Synopsis	*(Continue on the reverse side if needed)* * If H and K are two subgroups of a group G, then HK is a subgroup of G iff $HK = KH$ $HK = (HK)^{-1} = K^{-1}H^{-1} = KH$ $(HK)(HK)^{-1} = (HK)(K^{-1}H^{-1}) = H(KK^{-1})H^{-1} = HKH^{-1}$ $= KHH^{-1} = KH = HK$

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* If H_1 and H_2 are two subgroups of a group G then $H_1 \cap H_2$ is a Subgroup of G

* The union of two subgroups of a group need not be a subgroup of the group.

For example, $H_1 = \{0, 3\}$ and $H_2 = \{0, 2, 4\}$ are subgroups of Z_6

but $H_1 \cup H_2 = \{0, 2, 3, 4\}$ is not a Subgroup of Z_6

because $2 + 3 = 5 \notin H_1 \cup H_2$

* The union of two subgroups of a group is a subgroup iff one is contained in the other.

* All the subgroups of an abelian group are abelian

Teaching Plan / Lesson No.

Name of the Topic	Cosets and Lagrange's theorem
Hours required	5
Learning Objectives	After completion of this topic, students will be able to define the coset, discuss some properties of cosets
Previous knowledge to be reminded	Subgroups
Examples / Illustrations	For $H = \{3n \mid n \in \mathbb{Z}\}$, $0+H$, $1+H$ and $2+H$ are the only distinct left cosets
Additional inputs	Applications of Lagrange's theorem
Teaching Aids used	Blackboard, Chalkpiece
References cited	Topics in Algebra By I.N. Herstein
Student Activity Planned after the teaching	slip test
Activity planned outside the class room, if any	
Any other activity	
Topic Synopsis	*(Continue on the reverse side if needed)* Coset: Let H be a subgroup of a group G and $a \in G$. Then the set $aH = \{ah \mid h \in H\}$ is called a left coset of H in G generated by a and $Ha = \{ha \mid h \in H\}$ is called a right coset of H in G generated by a

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* If H is any subgroup of a group G and $h \in G$ then $hH = H = Hh$

* If a, b are any two elements of a group G and H is any subgroup of G Then $Ha = Hb \Leftrightarrow ab^{-1} \in H$ and $aH = bH \Leftrightarrow a^{-1}b \in H$

* If a, b are any two elements of a group G and H is any subgroup of G then $a \in bH \Leftrightarrow aH = bH$ and $a \in Hb \Leftrightarrow Ha = Hb$

* Any two left (right) cosets of a subgroup are either disjoint or identical.

* Let H be a subgroup of a group G. Then there exists a bijection between any two left cosets of H in G.

* If H is a subgroup of a group G then there is one-to-one correspondence between the set of all distinct left cosets of H in G and the set of all distinct right cosets of H in G.

<u>Lagrange's Theorem</u>

The order of a subgroup of a finite group divides the order of a group.

Converse of the above theorem is not true

For example, Take $H = \{i, -i\} \subseteq G = \{1, -1, i, -i\}$

Then $o(H) = 2 \mid o(G) = 4$

but H is not a subgroup of G

Teaching Plan / Lesson No.

Name of the Topic	NORMAL SUBGROUPS
Hours required	15
Learning Objectives	After completion of this topic, students will be able to define normal subgroup and discuss some properties of normal subgroups
Previous knowledge to be reminded	Subgroups and Cosets
Examples / Illustrations	$H = \{1, -1\}$ is a normal subgroup of $G = \{1, -1, i, -i\}$
Additional inputs	Sylow Theorems
Teaching Aids used	Blackboard, chalkpiece
References cited	Topics in Algebra By I. N. Herstein
Student Activity Planned after the teaching	Assignment, Seminar, slip test
Activity planned outside the class room, if any	
Any other activity	
Topic Synopsis	*(Continue on the reverse side if needed)* Normal Subgroup: A subgroup H of a group G is said to be a normal subgroup of G if $xhx^{-1} \in H \ \forall x \in G, h \in H$ * A subgroup H of a group G is normal iff $xHx^{-1} = H \ \forall x \in G$

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* A Subgroup H of a group G is a normal subgroup of G iff each left coset of H in G is a right coset of H in G.

* A Subgroup H of a group G is a normal Subgroup of G iff the product of two right (left) Cosets of H in G is again a right (left) Coset of H in G.

* Every subgroup of an abelian group is normal.

* If G is a group and H is a Subgroup of index 2 in G then H is a normal Subgroup of G.

* The intersection of any two normal subgroups of a group is a normal Subgroup.

* A normal subgroup of a group G is commutative with every complex of G.

* If N is a normal subgroup of G and H is any subgroup of G then HN is a subgroup of G.

* If H is a subgroup of G and N is a normal subgroup of G then (i) $H \cap N$ is a normal subgroup of H (ii) N is a normal subgroup of HN

* If N, M are normal subgroups of G then NM is also a normal subgroup of G.

* If M, N are two normal subgroups of G such that $M \cap N = \{e\}$ then every element of M commutes with every element of N

Simple group: A group G is called simple if it has no proper normal subgroups.

* Every group of prime order is simple

* Let H be a normal subgroup of G then the set $G/H = \{Ha / a \in G\}$ is a group w.r.t coset multiplication defined as

$$(Ha)(Hb) = Hab \text{ for } a, b \in G$$

Teaching Plan / Lesson No.

Name of the Topic	Homomorphism, Isomorphism of Groups
Hours required	15
Learning Objectives	After completion of this topic, students will be able to find that given map is a homomorphism or not. And, learn some properties related to homomorphism
Previous knowledge to be reminded	Groups and normal Subgroups
Examples / Illustrations	$f: (\mathbb{Z},+) \to (\mathbb{Z},+)$ defined as $f(x) = x+1$ is not a homomorphism
Additional inputs	
Teaching Aids used	Blackboard, Chalkpiece
References cited	Topics in Algebra by I.N. Herstein
Student Activity Planned after the teaching	Slip test, Assignment and Seminar
Activity planned outside the class room, if any	
Any other activity	
Topic Synopsis	*(Continue on the reverse side if needed)* If $(G,*)$ and (G',o) are two groups. If $f: G \to G'$ is a function satisfying the condition $f(x * y) = f(x) \, o \, f(y)$ for $x, y \in G$ then f is called a group homomorphism * Kernel of a homomorphism f is defined as $\ker f = \{x \in G \mid f(x) = e'\}$ (e' is identity in G')

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* Every homomorphic image of a group is a group

* Every homomorphic image of an abelian group is an abelian group

* G, G' are two groups, e, e' are identity elements in G, G' respectively

Then $f(e) = e'$ and

$f(a^{-1}) = (f(a))^{-1}$ for $a \in G$

* $f: G \to G'$ is a group homomorphism then $\ker f$ is a normal subgroup of G

* $f: G \to G'$ is a group homomorphism then f is an epimorphism iff

$\ker f = \{e\}$

<u>Fundamental Theorem of Homomorphism of groups</u>

Let G, G' be two groups and $f: G \to G'$ be a homomorphism then

$G / \ker f \cong f(G)$

or

Every ~~A~~ homomorphic image of a group is isomorphic to some quotient group

Teaching Plan / Lesson No.

Name of the Topic	Differential Equations of first order and first degree
Hours required	15
Learning Objectives	After completion of this topic, students will be able to solve differential equations of first order and first degree such as linear, Bernoulli, exact and non-exact.
Previous knowledge to be reminded	Differentiation and Integration Formulae
Examples / Illustrations	$\frac{dy}{dx} + 2xy = x^2$ is a linear differential equation of first order in y
Additional inputs	Applications of differential equations
Teaching Aids used	Blackboard, Chalkpiece
References cited	Ordinary Differential Equations by M.D. Raisinghania
Student Activity Planned after the teaching	Slip test, Assignment and Seminar
Activity planned outside the class room, if any	
Any other activity	
Topic Synopsis	*(Continue on the reverse side if needed)* An equation of the form $\frac{dy}{dx} + Py = Q$ is called a linear differential equation of first order in y. General solution is $y\, e^{\int P(x)\,dx} = \int Q(x)\, e^{\int P(x)\,dx}\, dx + c$

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* An equation of the form $\frac{dx}{dy} + P(y)\, x = Q(y)$ is called linear differential equation of first order in x

General solution is $x e^{\int P(y)dy} = \int Q(y)\, e^{\int P(y)dy}\, dy + c$

* An equation of the form $\frac{dy}{dx} + Py = Qy^n$ $(n \in \mathbb{R}, n \neq 0$ and $n \neq 1)$ is called a Bernoulli's differential equation. This equation can be solved by converting it into linear differential equation.

Exact differential equation:

An equation $Mdx + Ndy = 0$ is said to be an exact differential equation if there exists a function $f(x,y)$ having continuous first partial derivatives such that

$$M = \frac{\partial f}{\partial x} \text{ and } N = \frac{\partial f}{\partial y}$$

* A necessary and sufficient condition for $Mdx + Ndy = 0$ to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

* Working rule for solving an exact differential equation

1. Compare the given equation with $Mdx + Ndy = 0$ and find out M and N. Then find out $\frac{\partial M}{\partial y}$ and $\frac{\partial N}{\partial x}$

2. Given equation will be exact if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

3. Integrate M partially w.r.t x, treating y as constant. Denote it by $\int^x M dx$

4. Integrate only those terms of N, which do not contain x, w.r.t y

5. The general solution of the given exact differential equation is

$$\int^x M dx + \int (\text{Terms of } N \text{ not involving } x)\, dy = c$$

Teaching Plan / Lesson No.

Name of the Topic	
Hours required	
Learning Objectives	
Previous knowledge to be reminded	
Examples / Illustrations	
Additional inputs	
Teaching Aids used	
References cited	
Student Activity Planned after the teaching	
Activity planned outside the class room, if any	
Any other activity	
Topic Synopsis	*(Continue on the reverse side if needed)* Integrating Factor Let $Mdx + Ndy = 0$ be not an exact differential equation. If $Mdx + Ndy = 0$ can be made exact by multiplying it with a suitable function $\mu(x, y) \neq 0$, then $\mu(x, y)$ is called an integrating factor of $Mdx + Ndy = 0$

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Methods to find integrating factors of $Mdx + Ndy = 0$

Method 1: By Inspection

$d(xy) = xdy + ydx$ $\quad d[\log(xy)] = \dfrac{xdy + ydx}{xy}$ $\quad d\left(\dfrac{x}{y}\right) = \dfrac{ydx - xdy}{y^2}$

$d\left(\dfrac{y}{x}\right) = \dfrac{xdy - ydx}{x^2}$ $\quad d\left(\dfrac{y^2}{x}\right) = \dfrac{2xydy - y^2dx}{x^2}$ $\quad d\left(\dfrac{x^2}{y}\right) = \dfrac{2xydx - x^2dy}{y^2}$

$d\left(\tan^{-1}\dfrac{y}{x}\right) = \dfrac{xdy - ydx}{x^2+y^2}$ $\quad d\left(\tan^{-1}\dfrac{x}{y}\right) = \dfrac{ydx - xdy}{x^2+y^2}$ $\quad d\left(\dfrac{e^x}{y}\right) = \dfrac{ye^x dx - e^x dy}{y^2}$

Method 2: If $Mdx + Ndy = 0$ is a non-exact homogeneous differential equation and $Mx + Ny \neq 0$ then $\dfrac{1}{Mx+Ny}$ is an integrating factor of $Mdx + Ndy = 0$

Working Rule to Solve $Mdx + Ndy = 0$

1. General equation is $Mdx + Ndy = 0$ . . . (i)

   observe $\dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x} \Rightarrow$ (i) is not exact

2. Observe M and N are homogeneous functions of same order

3. Find $Mx + Ny$ and observe it $\neq 0$. Then $\dfrac{1}{Mx+Ny}$ is an I.F of (i).

4. Multiply (i) with I.F to transform it into an exact equation of (i)

   $M_1 dx + N_1 dy = 0$ . . . (ii)

5. Solve (ii) to get the general solution of (i)

Method 3: If the equation $Mdx + Ndy = 0$ is of the form $y f(x,y)dx + x g(x,y)dy = 0$ and $Mx - Ny \neq 0$ then $\dfrac{1}{Mx-Ny}$ is an integrating factor of

$Mdx + Ndy = 0$

Teaching Plan / Lesson No.

Name of the Topic	Differential Equations of first order but not of first degree
Hours required	15
Learning Objectives	After completion of this topic, students will be able to solve differential equations (nonlinear)
Previous knowledge to be reminded	Differentiation and integration
Examples / Illustrations	$\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} + y = 0$ is a non linear first order equation
Additional inputs	Applications of differential equations
Teaching Aids used	Blackboard, Chalkpiece
References cited	ordinary and partial differential equations by M.D. Raisinghania
Student Activity Planned after the teaching	Assignment, Seminar and slip test
Activity planned outside the class room, if any	
Any other activity	
Topic Synopsis	*(Continue on the reverse side if needed)* * An equation of the form $f(x, y, p) = 0$, where p is not of first degree is called a differential equation of first order and not of first degree These equations can be divided into four types: (i) Solvable for p (ii) Solvable for x (iii) Solvable for y (iv) clairaut's equation

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Equations solvable for p

Let $f(x,y,p)=0 \rightarrow (1)$ be the given equation of first order and degree n

$\therefore (1)$ can be written as $p^n + P_1(x,y)p^{n-1} + \dots + P_n(x,y) = 0 \rightarrow (2)$

If (2) is solved for p, let n solutions be

$$p = f_1(x,y),\ p = f_2(x,y),\ \dots\ p = f_n(x,y) \rightarrow (3)$$

$\therefore$ (2) can be expressed in the form

$$[p - f_1(x,y)]\,[p - f_2(x,y)]\ \dots\ [p - f_n(x,y)] = 0 \rightarrow (4)$$

Solving each equation in (4), we get n solutions

$F_1(x,y,c_1)=0,\ F_2(x,y,c_2)=0,\ \dots\ F_n(x,y,c_n)=0$ for n equations respectively

Since $F_i(x,y,c_i)=0$ is solution to $p=f_i(x,y)$ for each i, $F_i(x,y,c_i)=0$ is also solution of (4)

$\therefore$ The solution of (1) is $F_1(x,y,c_1)\,F_2(x,y,c_2)\dots F_n(x,y,c_n)=0$

Since the equation (1) is of first order, the general solution should have only one arbitrary constant.

Taking $c_1 = c_2 = \dots = c_n = c$, the general solution of (1) is

$$F_1(x,y,c)\,F_2(x,y,c)\dots F_n(x,y,c)=0$$

Equations solvable for x

Let $f(x,y,p)=0$ be the given differential equation $\rightarrow (1)$

If the equation (1) cannot be split up into rational and linear factors and (1) is of first degree in x, then (1) can be solved for x.

(1) can be expressed in the form $x = F(y,p) \rightarrow (2)$

Differentiating (2) w.r.t y gives an equation of the form $\frac{1}{p} = g(y, p, \frac{dp}{dy}) \rightarrow (3)$

Since (3) is an equation in two variables p and y, it can be solved

Teaching Plan / Lesson No.

Name of the Topic	
Hours required	
Learning Objectives	
Previous knowledge to be reminded	
Examples / Illustrations	
Additional inputs	
Teaching Aids used	
References cited	
Student Activity Planned after the teaching	
Activity planned outside the class room, if any	
Any other activity	
Topic Synopsis	*(Continue on the reverse side if needed)* Equations solvable for y: Let $f(x,y,p)=0$ be the given differential equation $\rightarrow$ ① If ① cannot be resolved into two rational and linear factors and ① is of first degree in y, then it can be solved for y

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① can be expressed in the form $y = F(x, p) \rightarrow ②$

Differentiation of ② w.r.t x gives an equation of the form

$$p = g\left(x, p, \frac{dp}{dx}\right) \rightarrow ③$$

Since ③ is an equation in two variables p and x, it can be solved.

The solution of ① is $\psi(x, y, c) = 0$

Equations that do not contain x (or y)

If the equation is of the form $f(x, p) = 0$ and is solvable for p, it will give $\frac{dy}{dx} = \phi(x)$, which is integrable. But if it is solvable for x, then $x = F(p)$

~~which is~~ If the equation is of the form $f(y, p) = 0$ and is solvable for p, it will give $\frac{dy}{dx} = \phi(y)$, which is integrable. But if it is solvable for y, then $y = F(p)$

Clairaut's Equation

The differential equation of the form $y = xp + f(p)$ is called Clairaut's equation. This equation is solved by considering it as $y = f(x, p)$, solvable for y type.

Solution of Clairaut's equation:

Let the given equation be $y = xp + f(p) \rightarrow ①$

$$\Rightarrow \frac{dy}{dx} = x\frac{dp}{dx} + p + f'(p)\frac{dp}{dx} \Rightarrow p = [x + f'(p)]\frac{dp}{dx} + p$$

$$\Rightarrow (x + f'(p))\frac{dp}{dx} = 0 \Rightarrow \frac{dp}{dx} = 0 \Rightarrow p = c$$

∴ The general solution of Clairaut's equation ① is $y = xc + f'(c)$

Teaching Plan / Lesson No.

Name of the Topic	Higher order Linear Differential Equations — I
Hours required	15
Learning Objectives	After completion of this topic, students will be able to solve the higher order linear differential equations with constant coefficients when $Q(x) = e^{ax}$ or $\sin ax$ or $\cos ax$ or x^k
Previous knowledge to be reminded	Finding roots of the equations and differentiation and integration formulae
Examples / Illustrations	$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$ is a second order linear differential equation
Additional inputs	—
Teaching Aids used	Blackboard and Chalkpiece
References cited	ordinary and partial differential equations by M.D. Raisinghania
Student Activity Planned after the teaching	Assignment, Seminar and Slip test
Activity planned outside the class room, if any	
Any other activity	
Topic Synopsis	*(Continue on the reverse side if needed)* An equation of the form $a_n\frac{d^ny}{dx^n} + a_{n-1}\frac{d^{n-1}y}{dx^{n-1}} + \dots + a_1\frac{dy}{dx} + a_0 y = Q(x)$ where $a_0, a_1, \dots a_n, Q$ are continuous real functions in x is called a linear differential equation of order n.

M. [signature]

Signature of the Lecturer

* If $y=y_1(x), y=y_2(x), \ldots y=y_k(x)$ are any k solutions of the k^{th} order equation $f(D)y=0$ then $y=c_1y_1+c_2y_2+\cdots+c_ky_k$ ($c_1, c_2 \ldots c_k$ are constants) is also a solution of $f(D)y=0$

Case(i) When the auxiliary equation has real and distinct roots then the general solution of $f(D)y=0$ is $y=c_1e^{m_1x}+c_2e^{m_2x}+\cdots+c_ne^{m_nx}$

$(D^2-3D+2)y=0$

Auxiliary equation is $f(m)=m^2-3m+2=0$

$\Rightarrow (m-1)(m-2)=0 \Rightarrow m=1,2$

General Solution is $y=c_1e^{x}+c_2e^{2x}$

Case(ii) When Auxiliary Equation has some equal roots then G.S is $y=(c_1+c_2x)e^{mx}$

$(D^2-2D+1)y=0$

A.E is $f(m)=m^2-2m+1=0$

$\Rightarrow (m-1)^2=0 \Rightarrow m=1,1$

G.S is $y=(c_1+c_2x)e^{x}$

Case(iii) When A.E has only a pair of conjugate complex roots then G.S is $y=e^{ax}(c_1\cos bx+c_2\sin bx)$, $m=a\pm ib$

$(D^2+D+1)y=0$

A.E is $f(m)=m^2+m+1=0$

$\Rightarrow m=\frac{-1\pm\sqrt{1-4}}{2}=\frac{-1}{2}\pm\frac{i\sqrt{3}}{2}$

G.S is $y=e^{-x/2}\left(c_1\cos\frac{\sqrt{3}}{2}x+c_2\sin\frac{\sqrt{3}}{2}x\right)$

Teaching Plan / Lesson No.

Name of the Topic	
Hours required	
Learning Objectives	
Previous knowledge to be reminded	
Examples / Illustrations	
Additional inputs	
Teaching Aids used	
References cited	
Student Activity Planned after the teaching	
Activity planned outside the class room, if any	
Any other activity	
Topic Synopsis	*(Continue on the reverse side if needed)* If Q is any function of x defined on an interval I and α is a constant then a particular value of $\frac{1}{D-\alpha}Q$ is equal to $e^{\alpha x}\int Q e^{-\alpha x}\,dx$ $\frac{1}{(D-\beta)(D-\alpha)}Q = \frac{1}{D-\beta}\left[e^{\alpha x}\int Q e^{-\alpha x}dx\right]$ $= e^{\beta x}\int\left[e^{\alpha x}\int Q e^{-\alpha x}dx\right]dx$

M. [illegible]

Signature of the Lecturer

* If $Q = e^{ax}$ then $P.I = \frac{1}{f(D)} e^{ax} = \begin{cases} \frac{e^{ax}}{f(a)}, & f(a) \neq 0 \\ \frac{1}{\phi(a)} \frac{x^k}{k!} e^{ax}, & \phi(a) \neq 0 \text{ when } a \text{ is repeated } k \text{ times} \end{cases}$

P.I of $(D^2-5D+6)y = e^{4x}$ is

$$y_p = \frac{1}{D^2-5D+6} e^{4x} = \frac{1}{16-20+6} e^{4x} = \frac{e^{4x}}{2}$$

* If $Q = \sin bx$ or $\cos bx$ then

$$P.I = \frac{1}{f(D)} Q = \frac{1}{\phi(D^2)} Q = \frac{1}{\phi(-b^2)} Q, \quad \phi(-b^2) \neq 0$$

if $\phi(-b^2) = 0$ then $P.I = \frac{1}{D^2+b^2} \sin bx = -\frac{x}{2b} \cos bx$

$$P.I = \frac{1}{D^2+b^2} \cos bx = \frac{x}{2b} \sin bx$$

P.I of $(D^2-D-2)y = \sin 2x$

$$y_p = \frac{1}{D^2-D-2} \sin 2x = \frac{1}{-4-D-2} \sin 2x = -\frac{1}{D+6} \sin 2x$$

$$= -\frac{D-6}{D^2-36} \sin 2x = -\frac{D-6}{-40} \sin 2x = 2\cos 2x - 6\sin 2x$$

$(1+x)^{-1} = 1 - x + x^2 - x^3 + \dots$, $(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots$

$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots$ $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots$

* If $Q = x^k$ then $P.I = \frac{1}{f(D)} x^k$

P.I of $(D^2-3D+2)y = 2x^2$

$$y_p = \frac{1}{D^2-3D+2} 2x^2 = \frac{2}{2\left(1+\frac{D^2-3D}{2}\right)} x^2 = \left(1+\frac{D^2-3D}{2}\right)^{-1} x^2$$

$$= \left[1 - \frac{D^2-3D}{2} + \left(\frac{D^2-3D}{2}\right)^2\right] x^2 = x^2 - \frac{D^2x^2 - 3Dx^2}{2} + \frac{9D^2}{4} x^2$$

$$= x^2 - \frac{2-6x}{2} + \frac{9 \cdot 2}{4} = \frac{2x^2 - 2 + 6x + 9}{2} = \frac{2x^2+6x+7}{2}$$

Teaching Plan / Lesson No.

Name of the Topic	Higher order Linear Differential Equations – II
Hours required	15
Learning Objectives	After completion of this topic, students will be able to solve the higher order linear differential equations when $Q(x) = e^{ax}\sinh bx$ or $e^{ax}\cosh bx$ or $e^{ax}V$ $(V = x^k)$
Previous knowledge to be reminded	Finding roots of equations and differentiation and integration formulae
Examples / Illustrations	
Additional inputs	
Teaching Aids used	
References cited	
Student Activity Planned after the teaching	
Activity planned outside the class room, if any	
Any other activity	
Topic Synopsis	*(Continue on the reverse side if needed)* Working rule to find P.I of $f(D)y = Q$, $Q = e^{ax}V$ $P.I = \frac{1}{f(D)} e^{ax} V$ $= e^{ax} \frac{1}{f(D+a)} V$

M. Rajyalakshmi

Signature of the Lecturer

Solve $(D^2-2D+1)y = x^2e^{3x}$

Sol: $y_c = (c_1+c_2x)e^x$

$$y_p = \frac{1}{(D-1)^2}x^2e^{3x}$$

$$= e^{3x}\frac{1}{(D+3-1)^2}x^2 = e^{3x}\frac{1}{(D+2)^2}x^2$$

$$= e^{3x}\cdot\frac{1}{4}\cdot\left(1+\frac{D}{2}\right)^{-2}x^2$$

$$= e^{3x}\cdot\frac{1}{4}\left(1-\frac{D}{2}+\frac{D^2}{4}\right)x^2$$

$$= e^{3x}\left(\frac{x^2}{4}-\frac{2x}{8}+\frac{2}{16}\right)$$

General solution of given equation is $y = y_c + y_p$

$$\Rightarrow y = (c_1+c_2x)e^x + e^{3x}\left(\frac{x^2}{4}-\frac{x}{4}+\frac{1}{8}\right)$$

Working rule to find P.I of $f(D)y = Q$; $Q = xV$

$$P.I = \frac{1}{f(D)}xV = x\frac{1}{f(D)}V - \frac{f'(D)}{[f(D)]^2}V$$

Solve $(D^2+4)y = x\sin x$

Sol: Given equation is $(D^2+4)y = x\sin x$

$f(D) = D^2+4$

$\Rightarrow f(m) = m^2+4 = 0 \Rightarrow m = \pm 2i$

$y_c = c_1\cos 2x + c_2\sin 2x$

$$y_p = \frac{1}{D^2+4}x\sin x$$

$$= x\frac{1}{D^2+4}\sin x - \frac{2D}{(D^2+4)^2}\sin x$$

$$= x\frac{1}{-1+4}\sin x - \frac{2D}{(-1+4)^2}\sin x$$

$$= \frac{x\sin x}{3} - \frac{1}{9}2\cos x$$

General solution to the given equation is

$y = y_c + y_p$

$$\Rightarrow y = c_1\cos 2x + c_2\sin 2x + \frac{x\sin x}{3} - \frac{2\cos x}{9}$$

Teaching Plan / Lesson No.

Name of the Topic	Higher order Linear Differential Equations (non constant coefficients)
Hours required	15
Learning Objectives	After completion of this topic, students will be able to solve the higher order linear differential equations with non constant coefficients
Previous knowledge to be reminded	Differentiation and integration formulae
Examples / Illustrations	$x\frac{d^2y}{dx^2} + (x-2)\frac{dy}{dx} - 2y = x^3$ is a higher order equation with non-constant coefficients
Additional inputs	
Teaching Aids used	Blackboard, chalkpiece
References cited	ordinary and partial differential equations by M.D. Raisinghania
Student Activity Planned after the teaching	slip test, Assignment, Seminar
Activity planned outside the class room, if any	
Any other activity	
Topic Synopsis	*(Continue on the reverse side if needed)* An equation of the form $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x) = R(x)$ where $P(x)$, $Q(x)$ and $R(x)$ are real valued functions of x is called the linear equation of the second order with variable Coefficients

M. Jaya

Signature of the Lecturer

Rules to find a solution of $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$

Rule 1: $y = e^{ax}$ is a solution if $a^2 + Pa + Q = 0$

Rule 2: $y = x^m$ is a solution if $1 - P + Q = 0$

a $y = x$ is a part of CF if $P + Qx = 0$

a $y =$ working rule to solve $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R \rightarrow (1)$

1. Reduce the given equation to standard form as given in (1)

2. Either a part of C.F is given or it can be found by inspection in some given above. Let it be $y = u$

3. Let $y = uv$ the general solution of (1) where v = ...

4. Then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ and $\frac{d^2y}{dx^2} = v\frac{d^2u}{dx^2} + 2\frac{du}{dx}\frac{dv}{dx} + u\frac{d^2v}{dx^2}$

5. With these substitutions in (1), it take the form

$$\frac{d^2v}{dx^2} + P_1\frac{dv}{dx} = R_1 \text{, where } P_1 = P + \frac{2}{u}\frac{du}{dx} \text{ and } R_1 = \frac{R}{u}$$

6. Taking $\frac{dv}{dx} = V$, it becomes $\frac{dV}{dx} + P_1 V = R_1$ which is linear in V and x hence it can be solved for V

7. $V = \frac{dv}{dx}$ on integration gives v.

8. Then the required general solution $y = uv$

The End

M. [illegible]